

Spatial Consequences of School Closures

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Motivation

- Declining populations have led local governments to implement school consolidation policies. [Examples](#)
- Puerto Rico has implemented such a policy, closing 44% of public schools. [Possible closures](#)
- Implementation of closures is difficult, key considerations include:
 - 1 Class sizes
 - 2 Fixed costs
 - 3 School quality
 - 4 Commuting to school
- Might not consider if/how parents adapt to school closures.

⇒ This could lead to broader welfare implications.

Welfare implications of school closures

Considering the extent to which parental households adapt to school closures is important since:

- 1 Parents could move \implies affects residential markets.
- 2 Parents could switch to private school \implies affects school markets.
- 3 Parents could stay put \implies absorb associated commuting costs and reduced school options.

\implies Build a Quantitative Spatial Model to consider these factors.

Objective and model preview

- The ability of households to adapt in so many ways make it difficult to assess the effect of school closures.
- My paper uses a Quantitative Spatial Model to examine how residential populations and housing markets respond to schooling.
- Specifically, the model incorporates: Commuting costs, residential choices, housing markets, and schools.

Research question

Research Question

To what extent do residential choices and housing markets respond to changes in school access in Puerto Rico following closures?

Overview:

This paper:

- 1 Embed schools into a Quantitative Spatial Model.
- 2 Show how a sufficient statistics approach allows us to estimate how residential choices and housing prices respond to school access.
- 3 Estimate these elasticities.

Today:

- Go through points (1) and (2)
- Estimation roadmap and challenges.

Details on the PR Education System

- Puerto Rico Department of Education (PRDE) runs a unified school district serving the entire territory.
 - Open enrollment and higher poverty imply that barriers to schooling could be different from usual context.
 - PRDE serves a population where 57% of children live in poverty (Ladd & Rivera-Batiz, 2006).
 - 85% of fourth graders in PR do not demonstrate basic proficiency in mathematics (National Center for Education Statistics, 2019).
- About a quarter of students enroll in private schools by 2003. (Ladd & Rivera-Batiz, 2006).

Details on closure

- 669 schools (44%) were closed in an 11-year period (2007-2018). [Graph](#) [Closed schools](#)
 - And closures were widespread. [Map](#)
- Puerto Rico has lost over 11% of its population from 2010-2020.¹
- Has experienced a 32% drop in the population of primary school youths (5-14 year olds) from 2010-2018. (Bobonis et al., 2022)

Large scale of consolidations present a good context for a Quantitative Spatial Model.

¹U.S. Census Bureau, 2020

Existing literature and contribution

Spatial Models and Schools

Agostinelli et al. (2023); Hsiao (2023); **Pietrabissa (2024)**

→ Simplifies analysis. Nationwide context. Examine school closure program instead of school expansion. Single decision maker.

School Closures

Bobonis et al. (2022); Brummer (2014); Epple et al. (2018)

→ Use a spatial model to study broader welfare implications of closure.

Methods

Ahlfeldt et al. (2015), **Tsivanidis (2023)**

→ Embed schools into the Quantitative Spatial Model. Show how the sufficient statistic approach applies to this context.

Data

Puerto Rico Department of Education

Additional Variables

- Large panel of administrative data on students, teachers, and schools from the AY 2010-2022.
- 2.7 million student-year records, belonging to 650,000 unique students.
- 46,000 of these students were affected by school closures.

ACS data at tract and block group level:

- School age population by age bins and gender (ages 5-9, 10-14, 15-17)
- Number of parent and non-parent households
- Median Household Income
- Median Household Price
- Home ownership rates
- Wages at the county level from County Business Patterns (CBP)

Spatial Model Intuition



Spatial Model Intuition



Suggestive evidence - Parents have different residential patterns.

Island-wide

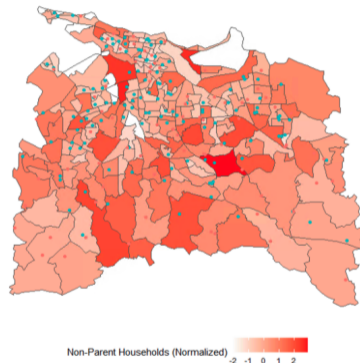
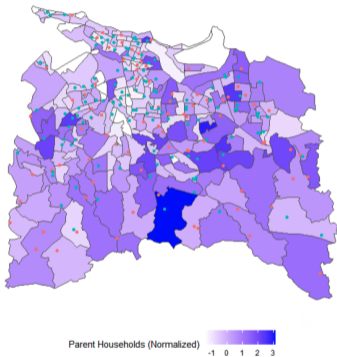


Figure: Distribution of parents households.

Figure: Distribution of non-parent households.

Embedding Schools into a QSM

- Household (ω) lives in a neighborhood n .
- Utility is a function of freely traded good (C_n), residential floorspace (H_n), neighborhood amenities (B_n).
- Households commute to work location (i) to earn their wages (w_i).
- Households also commute to a school (j) with school quality (S_j)
- Households pick residential and workplace location pair.
 - Preference shock over pair: ($b_{ni}(\omega)$)
 - Commuting cost: (τ_{ni}^W)
- Households pick schools.
 - Preference shock over school: ($s_j(\omega)$)
 - Commuting cost: (τ_{nj}^S)

A note on timing

- 1 Households choose residence-workplace pair ($n \rightarrow i$).
 - Take school access (S_n) into account when making this choice.
- 2 Households choose a school (j) after having chosen residence-workplace.

Embedding Schools into a QSM

Utility maximization leads to:

$$V_{nij} = \frac{\overbrace{B_n b_{ni}(\omega)}^{\text{Amenities}} \overbrace{w_i}^{\text{wages}}}{\underbrace{P_n^\alpha Q_n^{1-\alpha}}_{\text{Prices}} \underbrace{\tau_{ni} W}_{\text{Commute cost}}} \left(\frac{\overbrace{S_j s_j(\omega)}^{\text{School Quality}}}{\underbrace{\tau_{nj} S}_{\text{Commute cost}}} \right) \quad (1)$$

Second stage: School choice probabilities conditional on neighborhood

Assuming $(s_j(\omega))$ are Frechet distributed: Once you live on n , the probability of choosing school j :

$$\lambda_{nj|n}^S = Pr \left[V_{nj|n} > \max_{j' \neq j} V_{nj'|n} | \forall j' \neq j \right] = \frac{\overbrace{(S_j)}^{\text{benefit}} / \overbrace{(\tau_{nj}^S)}^{\text{cost}})^{\theta^S}}{\sum_{j'} (S_{j'} / \tau_{nj'}^S)^{1/\theta^S}} \quad (2)$$

Expected utility from schooling

The expected utility obtained from school in neighborhood n is proportional to access to high quality schools:

$$S_n = \mathbb{E}_j \left[\frac{S_j s_j(\omega)}{\tau_{nj}^S} \right] = \gamma^S \left[\left(\underbrace{\sum_j (S_j / \tau_{nj}^S)^{\theta^S}}_{\equiv \Phi_n^S} \right)^{1/\theta^S} \right] \quad (3)$$

$\Phi_n^S \equiv$ School Market Access for neighborhood n

\implies *This changes with school closures.*

First stage: Commuting probabilities

Assuming that preference shocks ($b_{ni}(\omega)$) are Frechet distributed, we obtain the standard probabilities for choosing a specific residence-commute pair (λ_{ni}^W):

$$\lambda_{ni}^W = Pr \left[V_{ni} > \max_{k,l} V_{kl}; \forall k, l \neq n, i \right] = \frac{\overbrace{(\mathbf{S}_n B_n w_i / \tau_{ni}^W P_n^\alpha Q_n^{1-\alpha})}^{\text{benefits}} \theta^W}{\sum_k \sum_l \overbrace{(\mathbf{S}_k B_k w_l / \tau_{kl}^W P_k^\alpha Q_k^{1-\alpha})}^{\text{costs}} \theta^W} \quad (4)$$

Residential probabilities

Summing over workplace locations gives us residential probabilities:

$$\lambda_n^R = \frac{R_n}{L_N} = \sum_l \frac{(\mathbf{S}_n B_n w_l / \tau_{nl}^W P_n^\alpha Q_n^{1-\alpha})^{\theta^W}}{\sum_k \sum_l (\mathbf{S}_k B_k w_l / \tau_{kl}^W P_k^\alpha Q_k^{1-\alpha})^{\theta^W}} = \frac{((\Phi_n^S)^{1/\theta^S} B_n)^{\theta^W} (P_n^\alpha Q_n^{1-\alpha})^{-\theta^W} \Phi_n^W}{\sum_k \sum_l ((\Phi_n^S)^{1/\theta^S} B_k)^{\theta^W} (P_k^\alpha Q_k^{1-\alpha})^{-\theta^W} \Phi_k^W} \quad (5)$$

Residential probabilities

Summing over workplace locations gives us residential probabilities:

$$\lambda_n^R = \frac{R_n}{L_N} = \sum_l \frac{(S_n B_n w_l / \tau_{nl}^W P_n^\alpha Q_n^{1-\alpha})^{\theta^W}}{\sum_k \sum_l (S_k B_k w_l / \tau_{kl}^W P_k^\alpha Q_k^{1-\alpha})^{\theta^W}} = \frac{((\phi_n^S)^{1/\theta^S} B_n)^{\theta^W} (P_n^\alpha Q_n^{1-\alpha})^{-\theta^W} \phi_n^W}{\sum_k \sum_l ((\phi_n^S)^{1/\theta^S} B_k)^{\theta^W} (P_k^\alpha Q_k^{1-\alpha})^{-\theta^W} \phi_k^W} \quad (5)$$

Where,

$$\phi_n^W = \sum_l (w_l / \tau_{nl})^{\theta^W} \quad (6)$$

Market Access Terms / Sufficient Statistics

As in Tsivanidis (2023), these Market Access terms are sufficient statistics for residential populations (R_n) and prices (Q_n). Where:

$$\phi_n^W = \sum_l (w_l / \tau_{nl})^{\theta^W} \approx \text{Household's access to wages from } n$$

$$\phi_n^S = \sum_j (s_j / \tau_{nj}^S)^{\theta^S} \approx \text{Household's access to schooling from } n$$

Endogenous outcomes as functions of Market Access

In the QSM with schools, residential populations (R_n) and prices (Q_n) in neighborhood (n) are functions of the Market Access terms.

$$\log \Delta R_n^W \approx \rho^W \log \Delta \Phi_n^W + \rho^S \log \Delta \Phi_n^S + e_n \quad (7)$$

$$\log \Delta Q_n^W = \rho'^W \log \Delta \Phi_n^W + \rho'^S \log \Delta \Phi_n^S + e'_n \quad (8)$$

Estimation roadmap

Now for estimation:

$$\phi_n^S = \sum_j S_j^{\theta^S} / e^{-\theta^S \kappa^S d_{nj}}$$
$$\phi_n^R = \sum_l w_l^{\theta^W} / e^{-\theta^W \kappa^W d_{nl}}$$

Estimation roadmap

$$\phi_n^S = \sum_j S_j^{\theta^S} / e^{-\theta^S \kappa^S d_{nj}}$$
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$$\phi_n^R = \sum_l w_l^{\theta^W} / e^{-\theta^W \kappa^W d_{nl}}$$

- 1 Wages (w_l) ← CBP data at the county level.

Estimation roadmap

$$\phi_n^S = \sum_j S_j^{\theta^S} / e^{-\theta^S \kappa^S d_{nj}}$$
$$\phi_n^R = \sum_l w_l^{\theta^W} / e^{-\theta^W \kappa^W d_{nl}}$$

- 1 Wages (w_l) ← CBP data at the county level.
- 2 θ^W ← Calibrate so that model wages match data for 2020.

Estimation roadmap

$$\phi_n^S = \sum_j S_j^{\theta^S} / e^{-\theta^S \kappa^S d_{nj}}$$
$$\phi_n^R = \sum_l w_l^{\theta^W} / e^{-\theta^W \kappa^W d_{nl}}$$

- 1 Wages (w_l) ← CBP data at the county level.
- 2 θ^W ← Calibrate so that model wages match data for 2020.
- 3 Transport costs between tracts (d_{nl}) ← GIS and Openstreet map.

Estimation roadmap

$$\Phi_n^S = \sum_j S_j^{\theta^S} / e^{-\theta^S \kappa^S d_{nj}}$$
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- 1 Wages (w_l) ← CBP data at the county level.
- 2 θ^W ← Calibrate so that model wages match data for 2020.
- 3 Transport costs between tracts (d_{ni}) ← GIS and Openstreet map.
- 4 Commuting elasticity ($\kappa^W \theta^W$) ← ACS commuting data for PR at county level (LODES data not available for PR).

Estimation roadmap

$$\phi_n^S = \sum_j S_j^{\theta^S} / e^{-\theta^S \kappa^S d_{nj}}$$
$$\phi_n^R = \sum_l W_l^{\theta^W} / e^{-\theta^W \kappa^W d_{nl}}$$

School Quality Estimation

Estimation roadmap

$$\Phi_n^S = \sum_j S_j^{\theta^S} / e^{-\theta^S \kappa^S d_{nj}}$$
$$\Phi_n^R = \sum_l w_l^{\theta^W} / e^{-\theta^W \kappa^W d_{nl}}$$

- 1 “Transformed” School quality ($S_j^{\theta^S}$) ← Calculate school quality from the model. Validate it with school data.

School Quality Estimation

Estimation roadmap

$$\Phi_n^S = \sum_j S_j^{\theta^S} / e^{-\theta^S \kappa^S d_{nj}}$$
$$\Phi_n^R = \sum_l W_l^{\theta^W} / e^{-\theta^W \kappa^W d_{nl}}$$

- 1 “Transformed” School quality ($S_j^{\theta^S}$) ← Calculate school quality from the model. Validate it with school data.
- 2 Transport costs (d_{nj}) ← GIS and Openstreet map.
- 3 Commuting elasticity ($\kappa^S \theta^S$) ← Use zip code data; otherwise use IPF IPF

School Quality Estimation

Some measurement challenges thus far:

- Have school locations, but only know student location at the zip-code level.
- Worker commuting and wage data is much more aggregated than US data, so can only accurately estimate (ϕ_n^W) at the county level.
- School quality measurement relies heavily on commuting totals.

Addressing endogeneity concerns

Need instrument for changes in School Access, as school closures could be driven by demographics (particularly, population loss). Endogeneity problems in Market Access traditionally addressed by:

- Evaluating changes in market access to locations not directly targeted by infrastructure change.
- Historical plans as an instrument: See if hypothetical changes Market Access an instrument.
- Geographic barriers.
- Donut instruments.

Addressing endogeneity concerns

Need to find some exogeneity in Market Access changes. An idea:

- Find schools which with low enrollment (likelihood of closure is high).
- Policy maker might not want to close all of these schools.
- However, if many of these schools are close, the policy maker might want to eliminate redundant schools.
- The choice of which schools to close at this level might be as good as random.
- Evaluate market access changes at this level with geographic fixed effects.

Challenges: Maybe not enough variation. Low power. Need to be careful with defining this set of schools.

To-do list

- 1 Estimate student commuting elasticity.
- 2 Estimate changes in Market Access.
- 3 Assess validity with robustness checks at various geographic levels and (hopefully) some instrument to school access.
- 4 Find other proper controls and check for robustness in my results.
- 5 Finish!

Thank you!

School closures in the US [back](#)

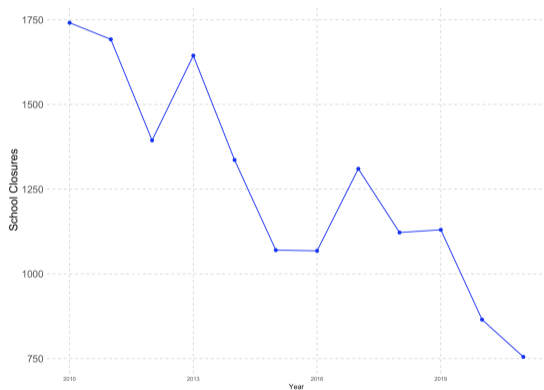


Figure: Number of schools closed each year in the US according (NCES).

School closures around the world [back](#)

121 Ontario schools slated to close: Report

People for Education report tallies schools on the chopping block, finds 'disproportionate impact' in rural areas.

May 4, 2017 3 min read 



Student Krish Gandhi shouts at passing cars during a rally to protest plans to close Robert Bateman High School in Burlington. Delegations are planning to appear before Halton trustees Monday to oppose the decision.

School closures around the world

[back](#)

More than 90 English primary schools to close or face closure for lack of pupils

Guardian analysis lays bare effect of dwindling pupil numbers and associated funding amid rising housing and childcare costs

- ['Where am I going to send my children?': anguish as schools close](#)



■ The analysis showed 88 primary schools in England were more than two-thirds empty last year, leaving them in danger of closure, with further four already proposed to close. Photograph: Chris Bull/Alamy

In the US... [back](#)

Puerto Rico has closed a greater proportion of public schools than any metropolitan region in the US in recent years.⁴

	Population ⁵ 2018, in millions	Student Enrollment 2018–2019	Number of closures	Percent closed in district
Philadelphia	1.584	202,767 ⁶	30 in 1 year (2013) ⁷	10%
Washington, DC	0.702	49,056 ⁸	38 in 6 years (2008–2014) ⁹	25%
Chicago	2.705	361,314 ¹⁰	200 in 16 years (2002–2018) ¹¹	33%
Puerto Rico	3.195	300,000 ¹²	673 in 11 years (2007–2018) ¹³	44%

”Better distribution of schools is needed.” [back](#)

 NOTICIA

Yanira Raíces sobre el cierre de escuelas: “La recomendación es hacer una mejor distribución”

La secretaria saliente de Educación ejemplificó que hay pueblos con una matrícula de apenas 600 alumnos que, si bien podrían ubicarse en un mismo plantel, están distribuidos en cinco

3 de diciembre de 2024 - 4:07 PM

↳ Actualizado el 3 de diciembre de 2024 - 5:46 PM

 COMPARTIR

 13

” More than 200 schools have less than 100 students.” [back](#)

CONTENIDO RELEVANTE

Más de 200 escuelas tienen una matrícula de menos de 100 estudiantes

La secretaria del Departamento de Educación reveló que cientos de planteles tienen menos de 10 estudiantes por aula

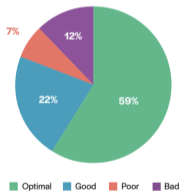
Andrea Cruz, EL VOCERO 03/12/2024 Actualizado hace Dec 3, 2024 

Current state of schools [back](#)

FIGURE 9

Current Physical Conditions of Vacant School Buildings

Out of 82 vacant school buildings visited, most (48) are in “optimal” condition. Nevertheless, almost a fifth (19) are either “poor” or “bad.”

**FIGURE 8**

Dilapidation at Vacant Schools

Out of 82 visited vacant schools, there was a variety of observable cases of dilapidation or potential risks.



Evidence on Consolidation - Bobonis, Sotomayor, & Wagner (2022) [back](#)

Uses Event Study approach to study the impact of school consolidation in this setting.

Main findings include:

- Overall effects of consolidation are modest - although there is considerable heterogeneity.
- Large re-sorting effects. Children displaced near a high achieving school see a 0.38 sd increase in performance after four years; students displaced from high achieving schools see a 0.21 sd decrease in performance.
- 1.2 p.p decrease in enrollment following closure.
- Learning gains from resorting are more likely to be achieved if an open school was nearby.
- Some evidence of positive peer effects.

These findings suggest that retaining access to high quality schools should be a key

Data from the PR Education Dept [back](#)

- School location
- Student residential zip code
- Standardized testing data (Pruebas Puertorriqueñas de Aprovechamiento Académico) from grades 3-8.
- Academic history from grades 3-8.
- Gender, age, poverty status, urban/rural, special education status.

44% of schools were closed in the last decade [back](#)

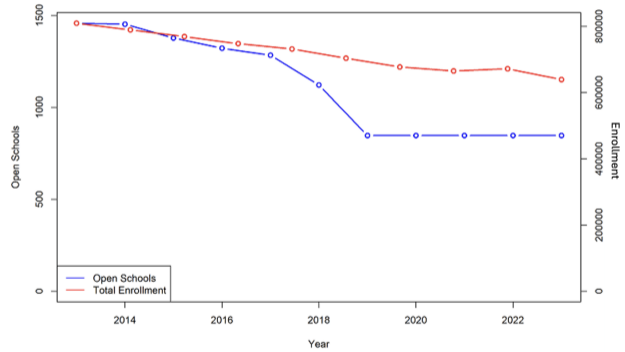


Figure: Number of total operating schools and total student from 2013-2023.

Map of closures

Every year

back

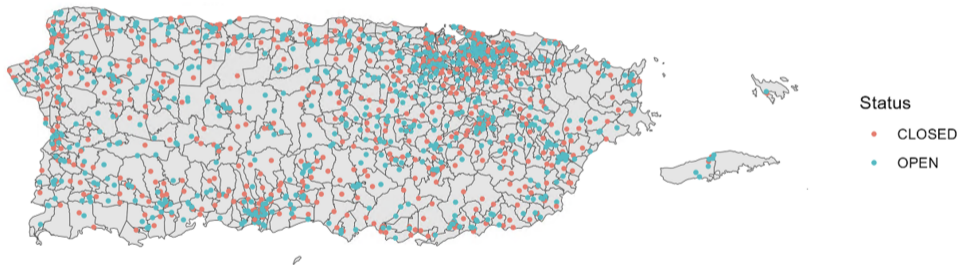


Figure: Operating status of public schools in Puerto Rico in 2023 by Census tract. Open schools are in blue and closed schools in red.

Island-wide distribution of parents vs non-parents

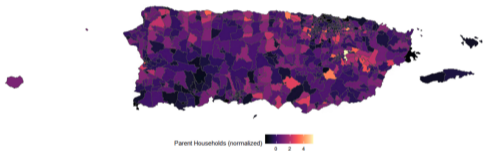


Figure: Parents households in PR

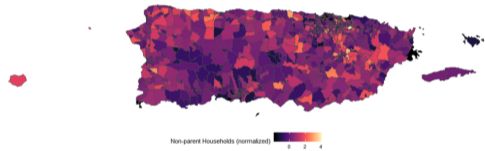


Figure: Non-parent households in PR

back

School closures over time

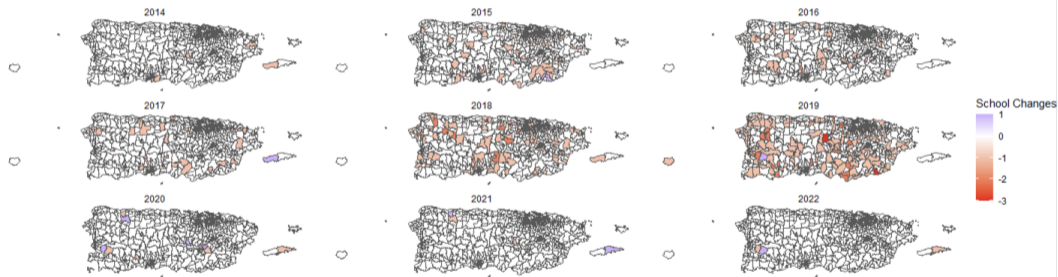
[back](#)

Figure: Closures of school throughout the island from 2014-2022.

Types of household

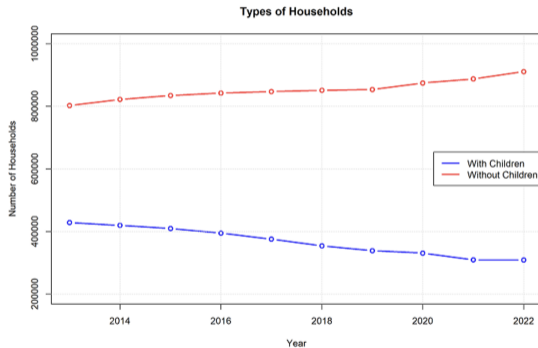


Figure: Types of households throughout time (5-year ACS estimates).

Main outcomes throughout time

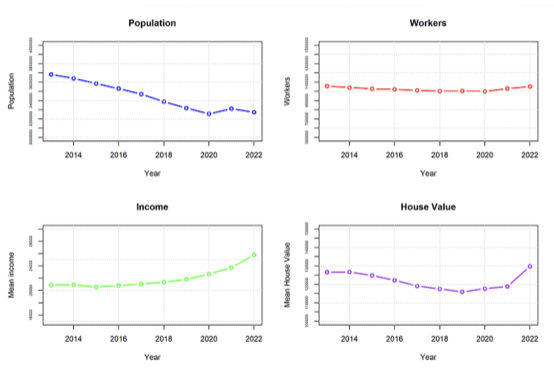


Figure: Evolution of demographic variables across time in Puerto Rico from 2013-2023 (5-year ACS estimates).

Workplace probabilities and Firm Market Access

Summing over residences give us workplace probabilities:

$$\lambda_i^L = \frac{L_i}{L_N} = \sum_k \frac{(S_n B_n w_i / \tau_{ni}^W P_n^\alpha Q_n^{1-\alpha})^{\theta^W}}{\sum_k \sum_l (S_k B_k w_l / \tau_{kl}^W P_k^\alpha Q_k^{1-\alpha})^{\theta^W}} = \frac{(w_i)^{\theta^W} \phi_i^F}{\sum_k \sum_l (w_l)^{\theta^W} \phi_l^F} \quad (9)$$

Firm Market Access is given by:

$$\phi_n^F = \sum_k (B_k S_k / \tau_{kl}^W P_k^\alpha Q_k^{1-\alpha})^{\theta^W} \approx \text{Firm's access to amenities} \quad (10)$$

back

Production

A production side is needed to close the model and determine wages and floorspace prices in equilibrium.

$$Y_n = A_n \left(\frac{L_n}{\beta} \right)^\beta \left(\frac{H_n^L}{1-\beta} \right)^{1-\beta} \quad (11)$$

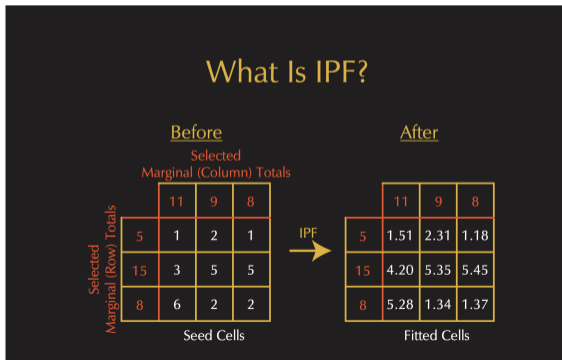
This gives us:

$$w_n = A_n \left(\frac{\beta}{1-\beta} \frac{H_n^L}{L_n} \right)^{1-\beta} \quad (12)$$

$$q_n = A_n \left(\frac{1-\beta}{\beta} \frac{L_n}{H_n} \right)^\beta \quad (13)$$

Iterative Proportional Fitting

Given school and county totals for certain age groups, we can try to estimate origin-destination flows by using Iterative Proportional Fitting:



Iterative Proportional Fitting

Can potentially improve procedure by:

- Setting implausible origin-destination pairs to (near) zero.
- Using data along many dimensions (gender, age, etc).

back

Estimating w^{θ^W}

We can use a similar approach to Ahlfeldt et al., 2015 to identify find “adjusted wages” (w^{θ^W}). For example, residential market clearing implies:

$$L_i = \sum_n \lambda_{ni|n} R_n = \sum_n \frac{(w_i^{\theta^W} / e^{-\theta^W \kappa^S d_{ni}})}{\sum_l (w_l^{\theta^W} / e^{-\theta^W \kappa^W d_{nl}})} R_n \quad (14)$$

Knowing $-\theta^W \kappa^W$, you can solve for a vector of “adjusted” wages (w^{θ^W}) above knowing the residential and workplace totals.

Gravity estimation of commuting parameters

As is standard for the literature, we can estimate $-\theta^W \kappa^W$ from a gravity regression implied by commuting probabilities (4):

$$\log(\lambda_{ni}) = -\theta^W \kappa^W d_{ni} + FE_n + FE_i + \epsilon_{ni} \quad (15)$$

Knowing $w_i^{\theta^W}$ from (14) and $-\theta^W \kappa^W$ is enough to calculate commuter market access (at county level):

$$\Phi_n^R = \sum_l (w_l / e^{-\kappa^S d_{nl}})^{\theta^W}$$

Gravity estimation of commuting parameter ($\kappa^W \theta^W$)

Table 2: Commuting gravity equation for workers.

Dependent Variable: Bilateral Commuting Probability	
Travel Time (Minutes)	-0.0896*** (0.0027)
<i>Fixed-effects</i>	
Origin County	Yes
Destination County	Yes
<i>Fit statistics</i>	
Observations	6,084
Squared Correlation	0.94089
Pseudo R ²	0.48522
<i>Clustered (Origin County) standard-errors in parentheses</i>	
<i>Signif. Codes: ***: 0.01, **: 0.05, *: 0.1</i>	

Note: Estimates are obtained using the Pseudo-Maximum Likelihood regression (PPML). Travel times are obtained from OpenStreet Map software and reflect travel time from county to county by car in minutes.

2

²Very similar to the estimate in Pietrabissa (2024) for high skill workers in Madrid (-0.089) Yay!   22/27

Comparing estimated wages to real data

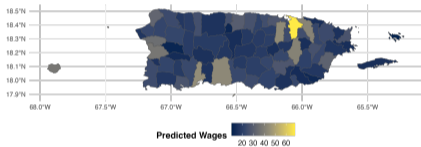


Figure: Predicted wages from model commuter market clearing condition (2020).

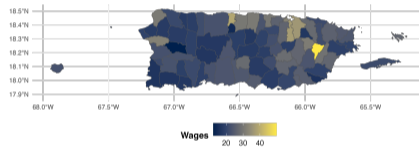


Figure: Average wages per county in CBP data (2020).

Comparing estimated wages to real data

Table 3: Fit between predicted wages from the model and wages observed in the data.

Dependent Variable:	Average Observed Wage
Constant	16.60*** (1.740)
Predicted wages	0.2851*** (0.0704)
<i>Fit statistics</i>	
Observations	78
R ²	0.17736
Adjusted R ²	0.16654

IID standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Note: Wages are obtained at the county level using the commuting market clearing condition for workers.

Comparing estimated wages to real data

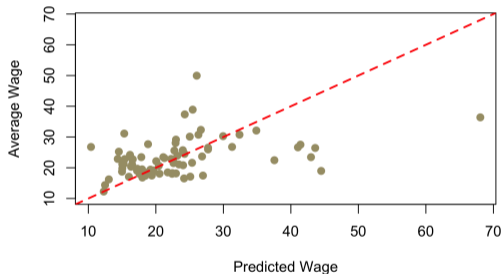


Figure: Comparing predicted wages to observed wages in the 2020 CBP data.

Obtaining school quality through residential and school student totals

An analogous condition holds for school choice:

$$E_j = \sum_n \lambda_{nj|n} R_n = \sum_n \frac{(S_j^{\theta^S} / e^{-\theta^S \kappa^S d_{nj}})}{\sum_{j'} (S_{j'}^{\theta^S} / e^{-\theta^S \kappa^S d_{nj'}})} R_n \quad (16)$$

Here, we observe enrollment totals for schools (E_j) and student residential totals (R_n). Given parameters $(-\theta^S \kappa^S)$, we could obtain (S^{θ^S}) that rationalize the totals we observe.

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Obtaining school parameters

- We propose using Iterative Proportional Fitting (IPF) to estimate a plausible commuting matrix for students, using our zip code data to inform the estimation procedure. **IPF**
- With an estimate of school commuting, we can use another gravity regression to obtain $-\theta^S \kappa^S$.

$$\log(\lambda_{nj|n}) = -\theta^S \kappa^S d_{nj} + FE_j + \epsilon_{nj} \quad (17)$$

This is enough to characterize School Market Access.

$$\Phi_n^S = \sum_j (S_j / e^{-\kappa^S d_{nj}})^{\theta^S}$$